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Recent results of estimating nonlinear time series models

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***Abstract*—**

THE main goal of this study is devoted to the problem of estimating VAR representations of multivariate (nonlinear) processes. In particular, we are interested to study the procedure of estimate and its asymptotic properties of VAR model under the assumption that the errors are uncorrelated but not necessarily independent. In this work, we introduce the main notions of linear and nonlinear process and briefly overviews results concerning the asymptotic behavior of the Quasi Maximum Likelihood Estimator (QMLE) in the weak AR framework. A particular attention is given to the estimation of the asymptotic variance matrix. Lagrange Multiplier, Wald and Likelihood Ratio tests are proposed for testing linear restrictions on the parameters of weak VAR model.

Keywords*—*VAR models, Nonlinear processes, QMLE, Lagrange Multiplier test, Wald test, Likelihood Ratio test.

I. Introduction

THe VAR model is a just a multiple time series generalization of autoregressive model (AR) in multivariate case. It was proposed by Sims (1980). This is one of the most successful and flexible model to analyze multivariate time series. Because, the VAR model have many advantages. First, it can explain a variable over its lags and according to the information contained in other variables, this raises the cointegration problems. Second, it has a very large information space. This method is simple to include the estimation procedures and tests. Its simplicity is due to the fact that there is no distinction between endogenous variables and exogenous variables, all variables are considered as endogenous. Also, with a VAR model it is possible to test causality relationships. So this model allows us to explore the dynamic relationship between several variables. In a VAR model, all variables are treated symmetrically.

In recent years, taking into account the nonlinearity tends to modify econometrics approaches applied to macroeconomic and finance. As we know that the most economic and financial time series exhibit nonlinear dynamic, regime switching and asymmetries. However, it is impossible to account for these phenomena from the usual type autoregressive ARMA or VAR linear models, we must have recourse to non-linear processes able to reproduce these features.

Among the great diversity of stochastic models for time series, it is customary to make a sharp distinction between linear and non-linear models. In fact, these two classes are not incompatible and can even be complementary. For a linear model to be quite general, the error terms must be the linear innovations, which are uncorrelated by construction but are not independent, nor martingale differences. In order to give a precise definition of a linear model and of a nonlinear process, first recall that by the Wold decomposition any purely non deterministic, second-order stationary process X_t can be represented by an infinite MA representation,

$$X_t = \sum_{l=0}^{\infty} \psi_l \xi_{t-l}, \tag{1}$$

where, $\sum_{l=0}^{\infty} \|\psi_l\|^2 < \infty$. This representation is weak because the noise is only the linear innovation of X_t (in the opposite case X_t would be a linear process). The process (ξ_t) is called the linear innovation process of the process $X = (X_t)$, and the notation $(\xi_t) \sim WN(0, \Sigma)$ signifies that $X = (X_t)$ is a weak white noise. A weak white noise is a stationary sequence of centered and uncorrelated random variables with common variance matrix Σ. By contrast, a strong white noise, denoted by $\xi_t \sim IID(0, \Sigma)$, is an independent and identically distributed sequence of random variables with mean 0 and variance Σ. A strong white noise is obviously a weak white noise, because independence entails uncorrelatedness, but the reverse is not true. Between weak and strong white noises, one can define a semi-strong white noise as a stationary martingale difference and is denoted by $\xi_t \sim MD(0, \Sigma)$, if ξ_t is a stationary sequence satisfying $E(\xi_t|\xi_u, u < t) = 0$ and $Var(\xi_t) = \Sigma$. We will see that the distinction between strong and weak white noises is fundamental in nonlinear time series analysis. The importance of nonlinear models has been more widely growing in the time series literature. These models are interesting and useful but may be hard to use. It is therefore of great importance to develop methods allowing to work with a broad class nonlinear time series models.

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This distinction has important consequences in terms of prediction. It is also crucial to take into account the differences between strong and weak VAR models. Thus, the class of the standard VAR models with independent errors is often judged too restrictive by practitioners, because they are inadequate for time series exhibiting a nonlinear behavior. On the contrary, the class of the so-called weak VAR models with uncorrelated but not necessarily independent errors is much more general and accommodates many nonlinear Data Generating Processes. The VAR model is called semistrong under the assumption a martingale-difference white noise. Obviously the strong VAR is more restrictive than that of semistrong VAR and the latter is more restrictive than the weak VAR. It is clear from these definitions that the following inclusions hold:

$$\{strongVAR\} \subset \{semi-strongVAR\} \subset \{weakVAR\}. \tag{2}$$

The class of the processes admitting weak VAR representations is dense in the set of the purely non deterministic stationary processes. Simple illustrations that the last inclusion of (2) is strict are given by the vast class of volatility models. Many examples of nonlinear processes admit weak AR representations, for exemple in the univariate case, the weak white noise (Romano and Thombs(1996)), causal representations of non causal strong AR models are weak AR (Francq, Roy and Zakoian (2005)). We conclude that the linear model (1), which consists of the MA models and their limits, is very general under the noise uncorrelatedness, but can be restrictive if stronger assumptions are maded.

Several papers in the recent time-series literature consider the problem of estimating and the asymptotic properties of general nonlinear models. We briefly review the most significant contributions. Romano et Thombs (1996), introduced uncorrelated and dependent process that can be extended to the multivariate case, the estimation of weak processes gave rise to many works. Dufour and Pelletier (2005) studied the asymptotic properties of a generalization of the regression-based estimation method proposed by Hannan and Rissanen (1982) under weak assumptions on the innovation process. Francq et al. (2005), point out a strong convergence of least squares estimators (LS) of ARMA, mainly as an ergodicity assumption on the observed process and asymptotic normality under the assumption of martingale difference noise. Thus, Boubacar and Francq (2009) extended these results to the case of a structural VARMA. They studied the asymptotic properties of quasi maximum likelihood estimators (QMLE) parameters of a VARMA model without the independence assumption on the noise. Francq and Rassi(2007) also provided evidence of the strong convergence of the LS estimator for vector autoregressive models, replacing the usual assumption difference martingale noise by mixing a hypothesis on the observed process and they studied portmanteau tests for weak VAR models. Rassi(2008) considered a non-stationary case of a VAR model with a weak white noise, it showed that the estimators of long-term relationships and the likelihood ratio test for the cointegrating rank have the same asymptotic behavior as in the standard case. Lin and McLeod (2007) considered an ARMA models with infinite variance. Patilea and Raissi (2010) extend these results to the case of a VAR model with time-dependent variance. They analyzed the VAR models when the innovations are unconditionally heteroscedastic. Francq and Zakoian (1998) proposed a variance estimator of the LS estimator for the ARMA representations of nonlinear processes, for which the difference martingale hypothesis is not verified. Thus, they showed a little convergence under the same assumptions. The technique used is HAC (Heteroscedasticity and Autocorrelation Consistent) method. Similarly, Francq and Rassi (2007) proposed a variance estimator of a weak VAR estimator by the spectral density (SP) method. Recently, Boubacar Mainassara et al. (2012) studied the problem of the estimate of the asymptotic variance.

The aim of the present work is devoted to the problem of estimating VAR representations of multivariate (nonlinear) processes. Thus, the principal interest is to study the procedure of estimate and its asymptotic properties. The traditional method consistently estimates the asymptotic covariance matrix of the parameter estimator and usually assumes the independence of the innovation process. For dependent innovations, the asymptotic covariance matrix of the estimator depends on the fourth-order cumulants of the unobserved innovation process, a consistent estimation of which is a difficult task. In the same way, we give a detailed attention to the problems of validation that is based on tests of linear restrictions on the parameters. Modified versions of the Lagrange Multiplier, Likelihood Ratio and Wald tests are proposed for testing linear restrictions on the parameters. The remainder of this work is organized as follows. Section 2 presents the notations and a preliminary asymptotics results. In the Section 3, we present the main results. Numerical illustrations are presented in Section 4. We finish by the conclusion.

II. Notations and preliminary asymptotic results

In econometrics and time series analysis, VAR models are much more widely employed to represent multivariate time series because they are easier to implement. Consider a d-dimensional stationary process (X_t) satisfying a vectorial AR model representation of the form:

$$X_t = \sum_{i=1}^{p} A_i X_{t-i} + \xi_t, \quad t \in Z. \tag{3}$$

It will be convenient to write (3) as $\phi(B)X_t = \xi_t$, where B is the backshift operator and $\phi(z) = I_d - \sum_{i=1}^{p} A_i z^i$ is the AR polynomial. The unknown parameter $\theta_0 = (A_{01}, ..., A_{0p})$ is supposed to belong to the interior of a compact subspace Θ of the parameter space $\Theta := \{\theta = (A_1, ..., A_p) \in R^p\}$. Since $\theta \in \Theta$, the polynomial $\phi(z)$ have all their zeros outside the unit disk. This assumption is standard and is also made for the usual strong AR models. The problem is obviously that the parameter θ_0 has to be estimated.

For the estimation of VAR models, the commonly used estimation method is the Quasi-Maximum Likelihood, which can also be viewed as a nonlinear least squares estimation. The asymptotic properties of the QMLE of VAR models are well-known under the restrictive assumption that the errors are independent (see $Lütkepohl(2005)$). Hannan and Deistler (1988) and Dunsmuir and Hannan (1976) studied the asymptotic behavior of the QMLE in a much wider context who proved consistency, under weak assumptions on the noise process and based on a spectral analysis. Thus, they have also obtained asymptotic normality under a conditionally homoscedastic martingale difference assumption on the linear innovations. However, this assumption precludes most of the nonlinear models.

The Gaussian quasi-likelihood is given by,

$$L_n(\theta)=\prod_{t=1}^{n}\frac{1}{\sqrt{2\pi}^n\sqrt{\det\Sigma}}\exp\{-\frac{1}{2}\xi_t^{'}(\theta)\Sigma^{-1}\xi_t(\theta)\}. \quad (4)$$

A quasi-maximum likelihood estimator is a measurable solution $\hat{\theta}_n$ of

$$\hat{\theta}_n=\arg\max_{\theta\in\Theta}L_n(\theta)=\arg\min_{\theta\in\Theta}l_n(\theta),\quad l_n(\theta)=-\frac{2}{n}\log L_n, \quad (5)$$

For the strong consistency and the asymptotic normality of the QMLE, it is necessary to assume the following assumptions:
$H_1:\xi_t\to WN(0,\Sigma)$.
H_2 : The process (ξ_t) is stationary and ergodic.
H_3 : We have $\theta_0\in\dot{\Theta}$ where $\dot{\Theta}$ denotes the interior of Θ.
H_4 : For some $V>0$, we have $E\|\xi_t\|^{4+2V}<+\infty$ and $\sum_{k=0}^{\infty}\{\alpha_\xi(k)\}^{\frac{V}{2+V}}<\infty$.
H_5 : La matrice $\dot{M}_{\theta_0}=\partial vec(M_\theta)/\partial\theta^{'}$ is of full rank, with $M_{\theta_0}=[A_{01}:...:A_{0p}:\Sigma_0]$. H_6 : For some $V>0$, we have $E\|\xi_t\|^{4+2V}<+\infty$ and $\sum_{k=0}^{\infty}\{\alpha_\xi(k)\}^{\frac{V}{2+V}}<\infty$.

The asymptotic distribution of the QMLE is given in the following proposition.
Proposition.1. Let (X_t) be a strictly stationary process defined by(3) satisfying A_1A_5. Then, $\hat{\theta}_n$ is QMLE and almost surely

$$\hat{\theta}_n\to\theta_0\quad a.s.\quad as\quad n\to\infty.$$

Proposition.2. Under the assumptions A_1-A_5, we have

$$\sqrt{n}(\hat{\theta}_n-\theta_0)\Rightarrow N(0,\Omega:=V^{-1}UV^{-1})\quad when\quad n\to\infty,$$

$$V(\theta_0)=\lim_{n\to\infty}\frac{\partial^2}{\partial\theta\partial\theta^{'}}l_n(\theta)=\lim_{n\to\infty}\frac{2}{n}\frac{\partial^2}{\partial\theta\partial\theta^{'}}\log L_n(\theta)$$

and

$$U(\theta_0)=\lim_{n\to\infty}Var\frac{\partial}{\partial\theta}l_n(\theta)=\lim_{n\to\infty}Var\frac{2}{\sqrt{n}}\frac{\partial}{\partial\theta}\log L_n(\theta).$$

In the standard strong VAR case we have $U=V$, so that $\Omega=V^{-1}$. In the general case we have $U\neq V$. The problem also holds in the univariate case (see Francq and Zakoan, 2007, and the references therein).

III. Main results

The asymptotic variance $\Omega:=V^{-1}UV^{-1}$ must however be estimated. The matrix V can easily be estimated by its empirical counterpart. So we need a consistent estimator of U. Note that

$$U=Var_{as}\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\Upsilon_t=\sum_{h=-\infty}^{+\infty}Cov(\Upsilon_t,\Upsilon_{t-h}), \quad (6)$$

$$\Upsilon_t=\frac{\partial}{\partial\theta}\{\log\det\Sigma+\xi_t^{'}(\theta^{(1)})\Sigma^{-1}\xi_t(\theta^{(1)})\}_{\theta=\theta_0}. \quad (7)$$

In the econometric literature the nonparametric kernel estimator, also called heteroscedastic autocorrelation consistent (HAC) estimator, is widely used to estimate covariance matrices of the form U. Let $\hat{\Upsilon}_t$ be the vector obtained by replacing θ_0 by $\hat{\theta}_n$ in Υ_t. The matrix is then estimated by a "sandwich" estimator of the form

$$\hat{\Omega}^{HAC}=\hat{V}^{-1}U^{HAC}\hat{V}^{-1},\quad U^{HAC}=\frac{1}{n}\sum_{t,s=1}^{n}\omega_{|t-s|}\hat{\Upsilon}_t\hat{\Upsilon}_s, \quad (8)$$

where $\omega_0,...,\omega_{n-1}$ is a sequence of weights.

It may be of interest to test s_0 linear constraints on the model. We thus consider a null hypothesis of the form

$$H_0:R_0\theta_0=r_0, \quad (9)$$

where R_0 is a known $s_0\times k_0$ matrix of rank s_0 and r_0 is a known s_0-dimensional vector. The Likelihood Ratio, Lagrange Multiplier and Wald test principles are employed frequently for testing H_0. We now examine if these principles remain valid in the non standard framework of weak VAR models.

The Likelihood Ratio statistic satisfies,

$$LR_n:=2\{\log L_n(\hat{\theta}_n)-\log L_n(\hat{\theta}_n^c)\}\overset{op(1)}{=}\frac{n}{2}(\hat{\theta}_n-\hat{\theta}_n^c)^{'}V(\hat{\theta}_n-\hat{\theta}_n^c). \quad (10)$$

The LR_n statistic asymptotically distribution as $\sum_{i=1}^{s_0}\lambda_iZ_i^2$ where the $Z_i's$ are iid $N(0,1)$ and $\lambda_1,...,\lambda_{s_0}$ are the eigenvalues of the matrice,

$$\Sigma_{\mathbf{LR}}=V^{-1/2}S_{\mathbf{LR}}V^{-1/2},\quad where \quad (11)$$

$$S_{\mathbf{LR}}=\frac{1}{2}(R_0V^{-1}R_0^{'})^{-1}(R_0\Omega R_0^{'})(R_0V^{-1}R_0^{'})^{-1}R_0. \quad (12)$$

Now, we consider the LM test. Let $\hat{\theta}_n^c$ be the restricted QMLE of the parameter under H_0. Define the Lagrangean,

$$\ell(\theta,\lambda)=l_n(\theta)-\lambda^{'}(R\theta-r), \quad (13)$$

where λ denotes a s_0-dimensional vector of LM. The first-order conditions yield,

$$\frac{\partial}{\partial\theta}l_n(\hat{\theta}_n^c)=R^{'}\hat{\lambda},\quad R\hat{\theta}_n^c=r_n. \quad (14)$$

Thus under H_0 and the previous assumptions,

$$\frac{1}{\sqrt{n}}\hat{\lambda} \rightarrow N\{0,(R_0V^{-1}R_0^{'})^{-1}R_0\Omega R_0^{'}(R_0V^{-1}R_0^{'})^{-1}\}, \quad (15)$$

so that modified versions of the Lagrange Multiplier statistic is defined by,

$$\begin{aligned} LM_n &= n\hat{\lambda}^{'}\{(R_0\hat{V}^{-1}R_0^{'})^{-1}R_0\hat{\Omega}R_0^{'}(R_0\hat{V}^{-1}R_0^{'})^{-1}\}^{-1}\lambda^{'} \\ &= n\frac{\partial}{\partial\theta}l_n(\theta_n^c)\hat{V}^{-1}R_0^{'}(R_0\hat{\Omega}R_0^{'})^{-1}R_0\hat{V}^{-1}\frac{\partial}{\partial\theta}l_n(\theta_n^c). \end{aligned}$$

More precisely, at the asymptotic level α, the null hypothesis is therefore rejected when $LM_n > \chi^2_{s_0}(1-\alpha)$.

According to the asymptotic normality of θ_0, we deduce that,

$$\sqrt{n}(R_0\hat{\theta}_n - r_0) \rightarrow N(0, R_0\Omega R_0^{'} := R_0(V^{-1}UV^{-1})R_0^{'}), \quad (16)$$

when $n \rightarrow \infty$. Under the assumptions A_1-A_5, the modified Wald statistic presented as follow:

$$W_n = n(R_0\hat{\theta}_n - r_0)^{'}(R_0\hat{\Omega}R_0^{'})^{-1}(R_0\hat{\theta}_n - r_0). \quad (17)$$

Under H_0, these statistics follow a distribution of $\chi^2_{s_0}$. Therefore, the standard formulation of the Wald test remains valid. At the asymptotic level α, the Wald test consists in rejecting the null hypothesis when $W_n > \chi^2_{s_0}(1-\alpha)$.

IV. Numerical illustrations

We illustrate the estimation procedure by applying it to some simulated sets. The first simulated example is used to demonstrate that the proposed procedure and the standard one give very close results when the underlying process is strongly linear. In the second example (a weak VAR representation of a non-linear process) our procedure outperforms the standard one.

We first study numerically the behaviour of the QMLE for strong and weak VAR(1) models of the form

$$X_t = AX_{t-1} + \xi_t, \quad A = 0.95I_2 \quad (18)$$

and VAR(2):

$$X_t = A_1X_{t-1}+A_2X_{t-2}+\xi_t, \quad A_1 = 0.95I_2 \quad et \quad A_1 = 0.1I_2 \quad (19)$$

where

$$\begin{pmatrix} \xi_{1t} \\ \xi_{2t} \end{pmatrix} \rightarrow \quad N(0, I_2) \quad (20)$$

in the strong case, and

$$\xi t = \begin{pmatrix} \eta_{1t}\eta_{1t-1}\eta_{1t-2} \\ \eta_{2t}\eta_{2t-1}\eta_{2t-2} \end{pmatrix}, \quad \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \end{pmatrix} iid \quad N(0, I_2), \quad (21)$$

in the weak case.

Table (I) displays the empirical sizes of the two versions (standard and modified) of the Wald, LM and LR tests. For the nominal level $\alpha = 5\%$, the empirical size over the $N = 1000$ independent replications should vary between the significant limits 3.65% and 6.35% with probability 95%. When the relative rejection frequencies are outside the significant limits, they are displayed in bold type in Table(I).

TABLE I. Empirical size of standard and modified tests: relative frequencies (in $\%$) of rejection of H_0. The number of replications is $N = 1000$.

Model	Length n	Standard Test			Modified Test		
		Wald	LM	LR	Wald	LM	LR
I	n=100	5.2	4.5	5.0	6.4	5.7	6.1
	n=500	5.6	5.4	5.5	6.0	5.8	5.9
	n=2000	5.8	5.8	5.8	5.8	5.7	5.7
II	n=100	**8.2**	**6.8**	**8.1**	4.9	3.9	4.8
	n=500	**7.9**	**7.7**	**7.8**	4.9	4.9	5.0
	n=2000	**6.5**	**6.5**	**6.5**	4.7	4.6	4.6

I: Strong VAR(1) model, II : Weak VAR(1) model

TABLE II. Empirical power of standard and modified tests : relative frequencies (in $\%$) of rejection of H_0. The number of replications is $N = 1000$.

Model	Length n	Standard Test			Modified Test		
		Wald	LM	LR	Wald	LM	LR
III	n=500	13.5	13.1	13.2	14.0	13.6	13.9
IV	n=500	8.4	9.11	11.3	22.1	24.0	27.2

III: Strong VAR(2), IV: Weak VARMA(2)

For the strong VAR model I, all the relative rejection frequencies are inside the significant limits. For the weak VAR model II, the relative rejection frequencies of the standard tests are definitely outside the significant limits. Thus the error of first kind is well controlled by all the tests in the strong case, but only by modified versions of the tests in the weak case.

From Table (II) we can remark that the powers of all the tests (standard and modified tests) are very similar in the model III case. We can also remark that the same is true for the three modified tests (Wald, LM and LR) in the model IV case. Whereas, the empirical powers of the standard tests are hardly interpretable for the model IV. Because we have already seen in Table (I) that the standard versions of the tests do not well control the error of first kind in the weak VAR framework.

V. Conclusion

In this study, we develop a review of some recent results for VAR models with uncorrelated but non independent errors. Replacing the usual implicit strong assumptions on the noise process by ergodicity and mixing modifies the asymptotic results. For the estimation of VAR models, the commonly used estimation method is the QMLE. It is known that, under the restrictive assumption that the errors are uncorrelated and independent, the QMLE are strongly consistent and asymptotically normal. We showed that these results can be used to estimate weak linear representations of some nonlinear processes.

In the same way, we give a detailed attention to the problems of validation that is based on tests of linear restrictions on the parameters. Modified versions of the Lagrange Multiplier, Likelihood Ratio and Wald tests are proposed for testing linear restrictions on the parameters.

From the asymptotic theory, we draw the conclusion that the standard approach, based on the quasi-maximum likelihood estimator, allows to fit vector autoregressive models of a wide class of nonlinear multivariate time series. In particular, this standard approach, including the significance tests on the parameters, needs however to be adapted to take into account the possible lack of the independence assumption.

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